Applying portfolio theory on the electricity sector: Installed capacity versus actual electricity generation

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Abstract
Portfolio theory has found its way in numerous applications for optimizing the electricity generation mix of a particular region. Existing models, however, consider typically a single time period and correspondingly do not properly account for actual dispatch constraints and energy sources with a variable output. This paper presents a portfolio-theory model that explicitly distinguishes between installed capacity (power), electricity generation (energy) and actual instantaneous power delivery. This way, the variability of wind power and the ramp limits of conventional power plants can be correctly taken into account in the investment optimization. The model is written as a quadratically constrained programming problem and used in a case study to optimize the Belgian generation mix.

Keywords: portfolio theory; electricity generation investment; wind power implementation

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1 Introduction
Policy makers have different tools at their disposal to set an appropriate framework in an attempt to guide power plant investments in a desired direction. When aiming for a long term supply strategy and generation mix, sufficient diversification of energy sources (fuels and technologies) is preferable. One way to quantitatively determine this diversification is by means of portfolio theory. Different electricity generation technologies and fuels are characterized by a certain cost, together with a standard deviation on that cost (risk). Correlations between different types of costs (e.g., investment costs, fuel costs, O&M costs) can be determined. Consequently, it is possible to define optimal portfolios, with minimum cost and/or risk levels.
The paper first presents a critical comprehensive overview of the most relevant literature concerning the application of portfolio theory in electricity planning. In the following section, a new portfolio modeling approach is developed, combining portfolio investment and actual dispatch decisions, in order to correctly account for the difference between installed capacity, energy delivery and actual instantaneous power delivery. This way, wind power can be included in the model. An application of the model (optimizing the Belgian generation mix), as an exemplary case study, is further presented. The final section concludes the paper.

2 Literature overview
The foundation of portfolio theory was laid by Markowitz in 1952 [1]. The basis of the theory states that by diversifying a portfolio of assets, the overall risk can be lowered compared to the risk of the individual assets. An early application of this theory to the electricity sector was presented by Bar-Lev and Katz [2].
The point of maximum return and the point of minimum risk are the extremes of the so-called efficient frontier. This frontier presents efficient portfolios, i.e., portfolios with minimum risk for a certain return or maximum return for a given risk.
Awerbuch and Berger [3] follow the basic portfolio approach using return (as an inverse of cost) to reflect upon an optimal generation mix for the EU. The authors assume a total amount of installed capacity (installed power). They test different scenarios and assume renewables to be riskless. The following costs are included in their model: investment costs, fuel costs and fixed and variable O&M costs. Other examples that follow this approach are presented in [4, 5]
A second model formulation is proposed by Jansen et al. [6]. The main difference of their approach with the above mentioned references lies in the fact that cost and cost risk are worked with, instead of return and return risk. Furthermore, energy instead of power is
used, this way accounting to some extent for the limited availability of renewable energy sources. Another example that follows this cost based approach is presented in [7]. Van Zon and Fuss [8] present the development of a vintage portfolio approach, using a single objective function. A total cost consisting of a weighted sum of the overall cost and the corresponding variance (risk) is minimized (using a risk-aversion factor). A distinction between installed capacity and electricity generation is also made, on a long-term time scale. Huang and Wu [9] elaborate further on this approach, also using a risk-weighted generation cost. They use a load duration curve to define different demand blocks. Gotham et al. [10] also present a portfolio approach, dividing load in different classes (having different load factors), this way accounting for the effects of different technologies operating in different classes with corresponding load factors. Doherty et al. [11] present a load duration based investment model focusing on wind penetration. Their model does not account for ramping issues. They also calculate the risk for different portfolios obtained with this model, but do not present an integrated portfolio theory based investment model.

The application of portfolio theory in a liberalized market environment is described by Roques et al. [12]. The expected net present value is worked with. Three base-load technologies are considered (nuclear, coal, CCGT), in three different scenarios (relating to different correlations between fuel, CO2 and electricity prices).

The approach presented in this work is a cost based approach, and considers portfolio optimization from an overall social standpoint (corresponding to some extent to the approach of Jansen et al. [6]). However, it extends the above mentioned references, as it encompasses an investment model, taking into account actual load patterns (hour-by-hour load), and corresponding dispatch issues as ramping constraints. By clearly distinguishing between investment in capacity (power, expressed in [MW]), electricity generation (energy, expressed in [MWh]) and actual instantaneous power delivery ([MWh/h] = [MW]), this model is able to correctly account for the variability of, e.g., wind power.

3 Model description

The most important feature of the developed approach is to make the distinction between installed capacity, electricity generation and instantaneous power delivery. The model itself decides upon the amount of power installed [MW], and also upon the generation, in [MWh], which is restricted by the installed capacity. This way, both costs and risks can be split up in a fixed part [€/MW], determined by the installed capacity, and in a variable part [€/MWh], determined by the electricity generated with certain power plant, during one hour.

The developed model has the structure of a quadratically constrained optimization problem (QCP). Wind power, having a variable profile is treated as a negative load and
subtracted from demand. Ramping constraints of classic power plants can be included in the model, this way accounting for the variability of wind power. The model is able to optimize the generation portfolio for a given load profile (typically one year).

The developed approach is described in two steps. A first step describes the integrated portfolio theory investment model. In a second step, wind power is introduced.

### 3.1 Integrated portfolio theory investment model

At first instance, the portfolio of minimum cost is determined. A distinction is made between fixed costs and variable costs. The fixed costs are expressed in terms of [€/MW]. These fixed costs consist of the investment cost \( INVi \) (annualized) and the fixed O&M cost \( FOMi \) (annualized). The fixed cost \( Fi \) of certain technology becomes:

\[
\forall i \in I : F_i = INVi + FOM_i
\]  

(1)

The variable costs \( vi \) consist of the fuel cost \( FU_i \) and the variable O&M cost \( VOM_i \), both expressed in terms of [€/MWh]:

\[
\forall i \in I : v_i = FU_i + VOM_i
\]  

(2)

The portfolio of minimum cost can be determined as follows. Considering one year, i.e., 8760 hourly load values, with \( I \) the set of available (conventional) technologies (index \( i \)) and \( J \) the set of time periods (index \( j \)), then the optimization problem becomes:

\[
\begin{align*}
\text{minimize cost} &= \sum_{i} F_i \cdot cap_i + \sum_{i,j} g_{i,j} \cdot v_i \\
\text{s.t.} & \quad \forall i \in I, \forall j \in J : \frac{g_{i,j}}{TP} \leq cap_i \\
& \quad \forall j \in J : \sum_{i} g_{i,j} = d_j
\end{align*}
\]  

(3)

(4)

(5)

with \( F_i \): annualized fixed cost (investment + fixed O&M) of technology \( i \) [€/MW]

\( v_i \): variable cost (fuel + variable O&M) of technology \( i \) [€/MWh]

\( TP \): length of the time periods considered, equal to 1 [h]

\( d_j \): demand for electricity during period \( j \) [MWh]

\( cost \): total cost of electricity generation (of one year) [€]

\( cap_i \): optimal installed capacity of technology \( i \) [MW]

\( g_{i,j} \): electricity generation of technology \( i \) during period \( j \) [MWh].

This formulation can be used to determine the optimal generation mix (installed capacities \( cap_i \)) with actual generation \( g_{i,j} \) and the corresponding cost. The outcome
corresponds to the mix determined by the well known load duration methodology [13]. However, by formulating the problem as a linear programming optimization problem, additional technical constraints as ramping limits can be included:

\[
\forall i \in I, \forall j \in J / \{1\} : g_{i,j} \leq g_{i,j-1} + \text{ramp}_i \cdot \text{cap}_i \cdot TP.
\]

\[
\forall i \in I, \forall j \in J : g_{i,j} \geq g_{i,j-1} - \text{ramp}_i \cdot \text{cap}_i \cdot TP
\]

with \( \text{ramp}_i \): relative ramping rate (both up and down) of technology \( i \) [-].

The average cost of this portfolio (expressed in [€/MWh]) is determined by:

\[
\text{avcost}_p = \frac{\text{cost}}{\sum_j d_j}
\]

The corresponding risk of this portfolio is determined as follows. A distinction is again made between the risk on fixed costs and the risk on variable costs. The risk on the fixed cost \( \sigma_{i,\text{fix}} \) consists of the risk on investment \( \sigma_{i,\text{INV}} \) and the risk on fixed O&M costs \( \sigma_{i,\text{FOM}} \), both expressed in [€/MW]:

\[
\forall i \in I : \sigma_{i,\text{fix}} = \sqrt{\sigma_{i,\text{INV}}^2 + \sigma_{i,\text{FOM}}^2}
\]

The risk on the variable cost \( \sigma_{i,\text{var}} \) consists of the risk on the fuel cost \( \sigma_{i,\text{FU}} \) and the risk on the variable O&M cost \( \sigma_{i,\text{VOM}} \), both expressed in [€/MWh].

\[
\forall i \in I : \sigma_{i,\text{var}} = \sqrt{\sigma_{i,\text{FU}}^2 + \sigma_{i,\text{VOM}}^2}
\]

Assuming a zero correlation between any cost components of the fixed cost and any other cost component of the variable cost (e.g., as in [6]), two separate correlation factors can be determined between the costs of two technologies \( i \) and \( h \), i.e., one for the fixed costs \( \rho_{\text{fix},ih} \) and one for the variable costs \( \rho_{\text{var},ih} \).

As the risks and correlation factors are decoupled between fixed and variable costs, the absolute risk \( \sigma_{p,\text{abs}} \) (expressed in [€]) of the portfolio can be written as follows:
The portfolio risk $\sigma_p$ in terms of [€/MWh] then becomes:

\[
\sigma_p = \sigma_{p,\text{abs}} / \sum_j d_j
\]  

(12)

Next to the portfolio of minimum cost, the portfolio of minimum risk can be determined. The objective function (3) therefore has to be replaced by a minimization function towards $\sigma_{p,\text{abs}}$ (11). The constraints remain the same (4) – (7). Using either the cost minimization or the minimization towards risk, and adding a constraint on the risk or cost, respectively, the efficient frontier is attained.

3.2 Accounting for wind power

It is further possible to add a certain amount of wind power to this system. To correctly account for the variability of this wind power, it is treated as a correction on the electricity demand (i.e., negative load). The constraint forcing supply equal to demand (5) now becomes:

\[
\forall j \in J : \sum_i g_{i,j} = d_j - wp_j \cdot cap_w
\]

(13)

with  $wp_j$: relative wind power profile, scaled to 1 [h]

$cap_w$: installed amount of wind power capacity [MW].

The cost function (3) is to be replaced with the following expression. Note that only fixed costs are considered:

\[
\text{cost} = \sum_i F_i \cdot cap_i + \sum_{i,j} g_{i,j} \cdot c_i + F_w \cdot cap_w
\]

(14)

with  $F_w$: annualized fixed cost (investment + fixed O&M) for wind power [€/MW].
The risk of wind power also has to be taken into account. Only fixed cost risks are considered. The risk on the total fixed cost $\sigma_{w,\text{fix}}$ of wind power can be composed from the risk on the investment cost $\sigma_{w,\text{INV}}$ and the risk on the fixed O&M cost $\sigma_{w,\text{FOM}}$ as follows:

$$\forall i \in I : \sigma_{w,\text{fix}} = \sqrt{\sigma_{w,\text{INV}}^2 + \sigma_{w,\text{FOM}}^2}$$

(15)

The correlation factor between the fixed costs of wind power and the fixed costs of any other conventional technology $i$ is denoted as $\rho_{\text{fix,wi}}$. The total risk (11) now has to be replaced by the following expression, accounting for the risk of wind power:

$$\sigma_{p,\text{abs}}^2 = \sum_i \sum_h \text{cap}_i \cdot \text{cap}_h \cdot \rho_{\text{fix,ih}} \cdot \sigma_{i,\text{fix}} \cdot \sigma_{h,\text{fix}} + 2 \cdot \sum_i \text{cap}_i \cdot \text{cap}_w \cdot \rho_{\text{fix,wi}} \cdot \sigma_{i,\text{fix}} \cdot \sigma_{w,\text{fix}}$$

$$+ \text{cap}_w^2 \cdot \sigma_{w,\text{fix}}^2 + \sum_i \sum_h \left( \sum_j g_{i,j} \right) \cdot \left( \sum_j g_{h,j} \right) \cdot \rho_{\text{var,ih}} \cdot \sigma_{i,\text{var}} \cdot \sigma_{h,\text{var}}$$

(16)

4 Simulation case study

4.1 Simulation set up

This section presents an application of the developed model on the Belgian power system. The actual Belgian load profile of 2008 and measured wind speed data are used. Due to computational restrictions, the model is not able to solve the QCP for a one year time frame (i.e., 8760 h). Therefore, an algorithm has been set up to select a number of typical days (covering the full load spectrum), with an average load equal to the yearly average load. The time frame used in the optimizations is 5 weeks (840 h). Results are scaled to a one year time frame.

Four conventional technologies are included, i.e., nuclear, coal, gas (CC) and oil (peak-units), all having specific ramp limitations (with nuclear the highest ramp restriction and peak oil the lowest). These technologies are complemented with wind power. The assumed costs are based on [14].

In this analysis, risks are determined from expertise and taken sufficiently diversified, as the goal of this paper is to illustrate the model and identify certain trends, rather than to focus on specific numerical outcomes. Results should therefore be interpreted with care. The portfolio optimization model as described in the previous section is now used to determine the portfolios of minimum cost and minimum risk, and the efficient frontier.
4.2 Simulation results

Three cases are discussed. The first is the reference case (data as presented above). A second example features a wind profile with a very high load factor (off-shore conditions), and is referred to as ‘high wind’ (the cost of wind remains unchanged). The third example attributes a higher risk to coal (in the framework of possible future stringent carbon restrictions).

Recall that this application is merely meant as an illustration of the model. As data result from generic engineering assumptions, results should be interpreted with care.

4.2.1 Reference case

At first instance, the model is used with the data described before, with wind data taken from a coastal on-shore location (load factor 29%). Figure 1 presents the efficient frontier, while Figure 2 presents the installed capacity of the various technologies in both the point of minimum cost and the point of minimum risk. Figure 3 presents the composition of the power system along the efficient frontier. The portfolio of minimum cost is dominated by nuclear power. Coal and gas\(^1\) follow second, and only a very limited amount of peak power is installed. No wind power is implemented in the portfolio of minimum cost. The cost for wind (together with the wind profile) makes it economically unattractive to install wind power. However, when moving along the efficient frontier, towards the point of minimum risk, wind power is gradually introduced in the generation mix, because of its relatively low risk.

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\(^1\) Note that the deployment of gas is limited. When applying portfolio theory in a liberalized market environment, gas CC is often the most attractive technology, due to the relatively low investment cost (and risk), and the high correlation between electricity and gas prices. However, the model developed and applied in this paper covers an entire generation mix optimization, from an overall (policy) social standpoint and as it is cost based, no electricity prices are considered (see literature overview).
Figure 1. Belgian electricity generation mix: portfolios of minimum cost and minimum risk and the efficient frontier.

Figure 2. Installed capacity by technology: portfolios of minimum cost and minimum risk.
Along the efficient frontier, the amount of coal fired power is significantly increased at the expense of nuclear, facing a high risk on investment. When wind power is introduced in the generation mix, this wind power is not able to replace any conventional capacity (as the employed wind profile has no wind at the moment of the highest load. The total capacity excluding wind power also increases, due to the need for additional rampable back up power (peak power), and due to the fact that units with a large number of operating hours (nuclear) are being replaced with units facing a lower load factor.

The relative electricity generation (Figure 4) again demonstrates the dominance of nuclear power in the minimum cost portfolio. Although the installed capacity of coal fired power plants in the portfolio of minimum risk is higher than the nuclear capacity, the amount of electricity generation with nuclear stays higher. The share of gas also diminishes due to the relatively high risk on the price of natural gas. Wind power contributes to about 10% of the electricity generation. Despite the relatively high amount of installed peak power, electricity generation remains below 2%, as this technology is only used sporadically to cover high fluctuations in the net demand (i.e., demand with wind energy subtracted).
As an example, Figure 5 presents the electricity generation by the different technologies during 5 selected days. The upper panel (a) presents the portfolio of minimum cost, while the lower panel (b) presents the portfolio of minimum risk. The increased use of coal and the implementation of wind are clearly reflected. When load is low and wind power is available, coal fired plants limit their output and are used in modulation.

Figure 4. Electricity generation (relative) by technology: portfolios of minimum cost and minimum risk.

Figure 5. Electricity generation by technology (expressed in \([\text{GWh/h}] = \text{[GW]}\)), in (a) the portfolio of minimum cost and (b) the portfolio of minimum risk.
4.2.2 **High wind**

In a second case, a wind profile of an off-shore location is applied (load factor 52%). The cost of wind power is, however, not changed (on-shore). This assumption could be motivated by lower costs through learning effects, or technology improvements and hence, a higher wind-to-power conversion. In this case, the portfolio of minimum cost encompasses already a certain amount of wind power, which increases further, to reach a level of 4.4 GW in the portfolio of minimum risk (Figure 6). The impact of the increased deployment of wind power is twofold, although with a similar outcome (i.e., the reduction of nuclear). First, in order to deal with the higher variability, the amount of rampable technologies increases at the expense of the hardly rampable nuclear power. Second, the net load is reduced, resulting in a smaller amount of base load. Note that there is also a reduction of nuclear due to the high risk on investment (independent of wind implementation).

The minimum installed capacity of conventional power along the efficient frontier is 13.1 GW (compared to 13.2 GW in the reference case). The employed high-wind profile is able to replace only a small amount of conventional power. The increase in conventional power with increasing wind power as observed in the reference case also occurs.

![Figure 6. Installed capacity by technology, in the portfolio of minimum cost and the portfolio of minimum risk, in the case of cheap wind power.](image)

The electricity generation in the portfolio of minimum cost is comparable to the reference case. In the portfolio of minimum risk, nuclear and coal now contribute with comparable amounts to the electricity generation. The use of gas is again reduced, while wind energy makes up more than 20% of the electricity generation. Figure 7 presents an example of
the electricity generation on the same 5 day period as in Figure 5. Coal and gas fired power plants are used for modulation, to deal with the large amount of wind energy and the corresponding fluctuations in net demand (panel b).

![Graph showing electricity generation by technology](image)

**Figure 7.** Electricity generation by technology, in (a) the portfolio of minimum cost and (b) the portfolio of minimum risk.

### 4.2.3 High risk coal

A third example is presented, attributing a high risk to coal (fuel), in the framework of possible stringent future carbon restrictions. The portfolio of minimum cost remains unchanged compared to the reference case. The portfolio of minimum risk, however, now faces a decrease in coal (due to the higher risk), balanced with an increase in both nuclear and gas fired power. Wind power also is introduced to some extent, while the deployment of peak power reduces, due to the availability of sufficiently rampable gas fired power.
Figure 8. Installed capacity by technology, in the portfolio of minimum cost and the portfolio of minimum risk, in the case of a high risk attributed to coal.

5 Conclusions

This paper presents an application of portfolio theory on the electricity generation mix, correctly accounting for the difference between installed capacity (power) and generated electricity (energy). The model itself determines the load factors of the different technologies installed. This approach allows to take variable energy sources like wind power into account (as it includes ramp-limits of different technologies). By including an actual wind profile (wind power is subtracted from demand, thereby resulting in a net load), a correct analysis can be made towards optimizing portfolios (both towards cost and risk).

An application on the Belgian power system with generic data is presented. In a first reference case, the portfolio of minimum cost consists of a large nuclear base load, complemented with coal and gas. Moving along the efficient frontier towards the point of minimum risk, gradually introduces wind energy in the portfolio. Scenarios with high wind and a high risk on coal in a carbon-constrained setting, are also discussed. A general conclusion concerning these examples is that lowering the overall risk can be a motivation for the implementation of wind power (at least to a certain extent). Correspondingly, this wind power implementation requires the deployment of additional rampable technologies (coal, gas and peak oil). The total installed capacity increases along the efficient frontier, as both wind power (which is not able to replace any capacity) and rampable back-up power increase.
The presented application demonstrates the functioning of the developed approach and corresponding model. It serves as a basis for further use. Restrictions (minima and/or maxima) on certain technology capacities can also be included. This allows to start from a certain (current) implementation, or to allow for political decisions (e.g., concerning nuclear power).
References